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Use of Three Parameter Model in Uniformity Trials experiments

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Abstract

Several linear and non-linear models have been proposed by various authors for the study of approximation of non linear models by statistical models which are linear in its parameters. Smith (1938), jessen (1942) and Hansen et.al. (1953), Misra (2010), Shukla et.al. 2018) have proposed various non-linear models for the comparative study of the parameters like coefficient of determination and sum of squired error. In this paper I have suggested an application of a three parameter linear model to compare these parameters of interest, which proves the suggested model a superior one.

Keywords- Linear model, non-linear model, three parameter model, variance function, reduction of error in modeling, coefficient of determination, sum of squired error.

Introduction

The application of the three parameter model is in the area of Design of Experiments concerning uniformity trials experiments. For explaining relationship between plot size and coefficient of variation, Fairfield-Smith (1938) arrived at the following empirical law

$$V = Ax^B \quad \textbf{(1)}$$

Where V is the coefficient of variation, x is plot size and A and B are constants. About the same time Mahalanobis (1940) developed the same relation in the studies on sample surveys in India. Robinson et.al. (1948) Warsom and Kalton (1953) used the relation **(1)** to uniformity trails data on peanuts and on brome grass respectively. The function **(1)** has been tremendously used in uniformity trails experiments. For further discussion and utility of uniformity trails experiments and use of function **(1)**, reference can be made of Federer (1995) and Panse and Sukhatme (1954).

The form **(1)** possesses all drawbacks as in case of determination of error term and as in other cases, the direct use of least squares method is not possible in this case also due to the term x^B in which the parameter B appears nonlinearly. Again, as in all previous cases, reciprocal term has been used in place of x^B, resulting into linear model of the form,

$$V = a + bX + \frac{c}{X} \quad , \quad X > 1 \quad \textbf{(2)}$$

Which is approximating adequately the form **(1)**.

The form **(2)** has been used in data sets reported in Haques et.al. (1988) and Weber and Horner (1957). The data sets of Haque et.al. (1988) consider relationship between plot size and coefficient of variation whereas data sets of Weber and Horner (1957) are on a study of optimum plot size and shape, which relate length of plot with variance of plot means per basic unit for yields in grams and also protein percentage. Table 1.1 reports results of data sets of Haque et.al. and Weber and Horner.

Result Table 1.1

Fitted curves, R^2 and s^2 values for the functional forms **(2)**

Source	Functional Forms					
			(2)			(1)
	A	b	c	R^2	s^2	s^2
Haque, et. Al. (1988),						
Table 1	20.9287	-1.1615	1.6239	0.9249	2.3256	2.88
Weber & Harner (1957)						
1. Yield, grams	472.7612	-31.5746	508.6567	0.9993	95.2910	29.15
2. Protein, %	0.0579	-0.0072	0.0659	0.9840	0.00004	0.00003

CONCLUSION:

It is seen from the table that linear form **(2)** fits reasonably well to data sets and s^2 values are comparable the form **(1)**. The estimates of b and c are either of opposite signs or b is close to zero.

Application of Three Parameter Model for Classical Freundlich Models and Modified Freundlich Models

The classical Freundlich (1926) model, with the deterministic component,

$$Y = \propto X^{\beta} \qquad \textbf{(3)}$$

is moderately close-to-linear nonlinear model (Ratkowsky, 1989). $\propto$ and β are parameters of model **(3)**. It belongs to the family of concave/convex curves that is, it has no maxima, minima or inflection points. The function **(3)** is nonlinear and the parameter appearing nonlinearly is β, However, form **(3)** is intrinsically linear and can be transformed through logarithmic transformation into a form, linear in its parameters, but such transformation presupposes a multiplicative error term and log normal distribution. The form **(3)** is of tremendous application in various areas of study, few of which are already mentioned in earlier studies, in which nonlinear functions are of type **(3)**. The form **(3)** also represents famous Cobb – Douglas function (Heady and Dillon, 1969), where variable X is resource measured, Y is output, $\propto$ and β are constants and constant β is also termed as elasticity of production. The model **(3)** is also one of the forms of Engel function.

As already mentioned in earlier studies, the term restricting direct use of least squares method is X^{β} and on the lines earlier studies, the linear model adequately approximating **(3)** up to moderate values of X, is,

$$Y = a + bX + \frac{c}{X} \quad , \quad X > 1 \qquad \textbf{(4)}$$

as usual a, b and c are parameters of the model **(4)**.

The model **(3)** has been applied to data sets reported in Nigam (1984) for studying relation between Income (as variable X) and substitution effects (as variable Y), in which he considered the form of Engel function like **(3)** as

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$$c = a\, I^{b} \qquad \textbf{(5)}$$

Where $c=c_i / c_s$, c_i is consumption of inferior commodity (money unite), c_s is consumption of superior commodity (money units), I is income of consumer and, a and b are constants. The form **(4)** has been used as an alternative approximation of (**5**). Table 1.2 reports the results of function **(4)** on data sets of Nigam (1984).

Result Table 1.2

Fitted curves, R^2 and s^2 values for the functional forms **(4)**

Source	**Functional Forms**					
			(4)			**(5)**
	a	b	c	R^2	S^2	S^2
Nigam (1984)	2.9517	-0.0037	112.8292	0.9808	0.0739	0.0798
	1.0565	0.00001	86.6752	0.9145	0.1556	0.2006

CONCLUSION:

It is seen from the table 1.2 that fit is satisfactory and model **(4)** describes the data well.

An extension of Freundlich model is modified Freundlich model,

$$Y = \propto\ + \beta X^{\rho} \qquad \textbf{(6)}$$

α, β and ρ are parameters of the model **(6)**. Ratkowsky (1989) remarked that model **(6)** is reasonably close-to-linear model, but it is intrinsically nonlinear so that it can not be transformed into form linear in its parameters.

In the form **(6)** if ρ is known, α and β can be easily estimated by direct application of least squares method. The term X^{ρ} bars the direct use of least squares technique. As in sections 3.1 and 3.2, we approximated X^{ρ} and used a reciprocal term in place of X^{ρ}. We also included a linear term on the basis of suggestions made by Shah and Khatri (1965). Thus the model, approximating **(6)** up to moderate values of X, is again of the form,

$$Y = a + bX + \frac{c}{X} \quad , \quad X > 1 \qquad \textbf{(7)}$$

The form **(7)** has been applied on data sets provided in Draper and Smith (1981), Box and Tidwell (1962) and Turmer et.al. (1959) and the results are reported in Table 1.3. Box and Tidwell (1962) arrived at conclusion to use the form

$$Y = \propto + \beta X^{1/2} \qquad \textbf{(8)}$$

to their data set and Turner et. Al. (1959) recommended use of the form,

$$\widehat{Y} = 5.04 + 18.34\, X^{-0.426} \qquad \textbf{(9)}$$

to their study of track records data. Both the forms **(8)** and **(9)** are modified Freundlich fitted models.

Result Table 1.3

Fitted curves, R^2 and s^2 values for the functional forms **(7)**

Source	**Functional Forms**					
			(7)			**(6)**
	a	b	c	R^2	S^2	S^2
Draper & Smith (1981)						
Ex. P, Pg. 525-525	63.3333	-1.1411	57.8309	0.9991	1.1097	7.323
	38.5529	-0.3993	51.1544	0.9968	2.6676	4.7659
Box and Tidwell			**(7)**			**(8)**
(1962)	39.5529	10.4632	-8.2961	0.9950	4.8543	1.84
Turner et. al.			**(7)**			**(9)**
(1959)	6.5866	0.3395	0.0758	0.9559	0.1123	0.023

FINAL CONCLUSION:

It is apparent from the table that fit is satisfactory. The functional forms **(6)**, **(8)** and **(9)** are fitted by ad hoc procedures. It is observed that the form **(7)** explains the data sets adequately.

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