

Evaluation Of Statistical Zero Inflated Single Sampling Plan Using Fuzzy Logic

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Abstract

Acceptance Sampling is a statistical technique used to determine the quality of a batch or lot of products by inspecting a random sample and ensuring products meets quality standards while minimizing inspecting costs. In a well-monitored manufacturing environment, non-conformities occur infrequently, resulting in many instances with zero non-conformities. Under such circumstances, the appropriate probability distribution for the number of non-conformities is a Zero-Inflated Poisson distribution. This paper describes in designing Single Sampling Plans by attributes when the number of non-conformities per product follows the Zero-Inflated Poisson distribution [ZIPD]. The design methodology focuses on the performance measures and OC curves are also provided. This paper focuses on designing of ZIPD using fuzzy logic theory and highlighting its efficiency with existing plans.

Keywords: Fuzzy Logic, OC Curve, Poisson distribution, SSP, Uncertainty, ZIPD.

Introduction

Acceptance sampling is a statistical quality control method used to determine whether to accept or reject a specific lot of products based on a sample. This method helps in making decisions about the quality of a batch without inspecting each and every item, which is particularly useful in mass production scenarios. Hence, acceptance sampling plans are a fundamental tool in quality control, helping organizations maintain product quality while optimizing inspection resources. By carefully designing and implementing a sampling plan, businesses can achieve a balance between quality assurance and operational efficiency based on the shop floor situation in any msd.

Acceptance sampling by attributes includes Single Sampling Plans (SSP), Double Sampling Plans (DSP), Multiple Sampling Plans, Sequential Sampling Plans, Continuous Sampling Plans, and Skip-Lot Sampling Plans and other special purpose sampling plan. SSP by attributes is characterized by two parameters: sample size (n), and acceptance number (c). The plan parameters are determined to protect the interests of both the producer and the consumer. The process of determining these parameters, ensuring protection for both parties, is termed “designing SSPs by attributes”. More details about determination of sampling plans can be found in Stephens (2001) and Schilling and Neubauer (2009). The procedure for determining the plan parameters based on the operating ratio and unity values under the conditions of Poisson distribution are discussed in Duncan (1986) and Schilling and Neubauer (2009).

Zero Inflated Poisson Distribution (ZIPD)

Due to the technological development, production processes are well designed in such a way that the products are in perfect state, so that the number of zero defects will be found more in those cases. However, random fluctuations in the production processes may lead some products to an imperfect state. The appropriate probability distribution to describe such situations is a zero-inflated Poisson (ZIP) distribution. The ZIP distribution can be viewed as a mixture of a distribution which degenerates at zero and a Poisson distribution.

ZIP distribution has been used in a wide range of disciplines such as agriculture, epidemiology, econometrics, public health, process control, medicine, manufacturing etc.

Review of Literature

In the following, provide a brief but comprehensive literature review to show how other researchers have used the ZIP distribution to model real-life data.

The ZIP distribution has been used as an appropriate probability distribution in diversified fields. Lambert et.al (1992) fitted a ZIP regression model to the data concerning the number of defects in a manufacturing process. Gurma et.al (1996) applied ZIP model for studying recreational trips. Saei et.al (1997) used this distribution for studying chemotherapy use. Bohning (1998) argues that the simple Poisson distribution is often unsuitable for datasets with numerous zeros. For example, in a study of 98 HIV-positive men, the number of urinary tract infections showed a large number of zero counts. When visualized, there is a significant spike at zero, and the Poisson model does not fit well. However, the ZIP model fits better. Therefore, Bohning suggests that the ZIP model is more appropriate when there is an inflation of zeros in count data. Ridout et.al (1998) have studied the problem of modelling count data with many zeros in horticultural research and investigated the appropriateness of the ZIP (Z_i, λ) model over the zero-inflated negative binomial model. Dankmar Bohning et.al (1999) have used ZIP (Z_i, λ) regression model to study a set of dental epidemiological data. Hall (2000) has carried out a case study on irrigation of greenhouse crops.

Xie et al., 2001 has described that Poisson distribution has often been used for count related data. He has explained with some examples. In a near zero-defect manufacturing environment, there are many zero-defect counts even for fairly large sample size. In this situation ZIPD is more appropriate. Xiang Liming et.al (2007) has discussed about the count data with extra zeros are common in many medical applications. A score test is projected for testing the ZIP mixed regression model against the Zero-inflated negative binomial alternative. Sampling distribution and power of the test statistic are evaluated by simulation studies. Traiquil Hasan et.al (2009) has proposed a non-stationary, observation driven time series model-based correlation structure. He has discussed about the estimation of the model parameters and the inefficiency of the estimators when the correlation structure is mis-specified. Zhao Yang et.al (2010a) has simplified the score statistic to test over dispersion in the zero-inflated generalized poisson (ZIGP) mixed model, and discussed an extension to test over dispersion in ZIP mixed models. Zhao Yang et.al (2010b) observed that the negative binomial (NB) model and the generalized poisson (GP) model are common to poisson models when over thinning out is present in the data. He has derived two score statistics from the GP model to zero-inflation. He has used simulation study illustrates that the developed score statistics reasonably follow a chi-square distribution and maintain the nominal level. Extension simulations also designate the power behavior is different including continuous variables than a binary variable in zero-inflation. Zhao Yang et.al (2010c) enlarged the score test statistic for over diffusion in poisson and binomial regression models to the zero-inflated poisson models. He has pointed out many examples which are related to ZIPD.

Jun Yang et.al (2011) described about the ZIPD which has been used in the modelling of count data in different contexts. This model tends to be prejudiced by outliers because of the excessive occurrence of zeros, thus outlier identification and robust parameter estimation are important for such distribution. To eradicate the effects of outliers, he has used two robust parameter estimates are proposed based on the trimmed mean and the winsorized mean. Loganathan et.al (2012) discussed the notion of single sampling plans by attributes in detail under the assumptions of a zero-inflated poisson distribution. The OC function of the single sampling plan under the conditions of ZIPD has resultant. A unity value under the condition of ZIPD has presented and tabulated for the SSP under ZIPD reduces both producer's risk and consumer's risk.

Uma et.al (2016) presented QSS with the reference to single sampling plan using ZIPD as the baseline distribution, and constructed the tables for various system indexed by various combinations of parameters for acceptance number tightening. Uma et.al (2016) discussed the determination of Quick Switching System by Attributes under the conditions of Zero-inflated Poisson Distribution. In this paper, the sample size and the acceptance numbers are calculated for the fixed AQL and LQL values. Raffie (2020) developed the Tightened-Normal-Tightened (TNT) sampling scheme by attributes, specifically for cases where the number of defects follows a zero-inflated Poisson (ZIP) distribution. Unity values were considered in constructing the scheme under these conditions. Numerical examples were provided to illustrate the determination of the TNT sampling scheme under the ZIP distribution and to evaluate its performance in comparison with the TNT scheme under the standard Poisson distribution.

Tajuddin et al. (2022) evaluated how well the zero–one-inflated Poisson–Lindley distribution models overdispersed data with an excess of zeros and ones by varying their proportions in the dataset. Their analysis of real-world datasets with these characteristics revealed that the proposed distribution outperforms other competing models, providing the best fit. Sun et al. (2023) introduced the zero-one-two-inflated Poisson (ZOTIP) distribution to model count data with excess zeros, ones, and twos. This distribution incorporates the zero-inflated Poisson (ZIP) and the zero-and-one-inflated Poisson (ZOIP) distributions as special cases. Abusaif et al. (2024) introduced a new flexible count regression analysis. They proposed an arbitrary multiply-inflated count regression model based on the modified Poisson distribution. To validate their model, they analyzed two practical datasets, demonstrating its superiority over existing alternatives.

Acceptance Single Sampling Plan with Fuzzy Parameter using Poisson Distribution

To inspect a large lot of size N , we first take a randomized sample of size n from the lot. We then inspect all items in the sample and count the number of defective items (d). If the number of defective items is less than or equal to the acceptance number, the lot will be accepted; otherwise, it will be rejected. When the lot size is large, the random variable ‘ d ’ follows a binomial distribution with parameters n and p , where p represents the proportion of defective items in the lot. However, if the sample size is large and p is small, the random variable ‘ d ’ can be approximated by a Poisson distribution with $\lambda = np$. The probability that the number of defective items is exactly ‘ d ’ and is given by,

$$P(d) = \frac{e^{-\lambda} \lambda^d}{d!} \dots\dots\dots (1)$$

And the probability of acceptance of the lot $P_a(P)$ is:

$$P_{a(P)} = P(d \leq c) = \sum_{d=0}^c \frac{e^{-\lambda} \lambda^d}{d!} \dots\dots\dots (2)$$

Suppose that we want to inspect a large lot of size N , with an unknown precise proportion of damaged items. We represent this parameter with a fuzzy number \tilde{p} as follows:

$$\tilde{p} = (a_1, a_2, a_3)$$

$$\text{where, } p \in \tilde{p}[1], q \in \tilde{q}[1], p + q = 1$$

A single sampling plan with a fuzzy parameter is defined by the sample size n , and acceptance number c . If the number of observation defective item is less than or equal to c , the lot will be accepted. For a large N , the number of defective items in this sample (d) follows a fuzzy binomial distribution. If \tilde{p} is a small, the random variable d follows a fuzzy Poisson distribution with parameter $\tilde{\lambda} = \tilde{n}\tilde{p}$. Thus, the fuzzy probability of having exactly d defective items in a sample is given below:

$$\tilde{p}(d - \text{defective})[\alpha] = [P^L[\alpha], P^U[\alpha]]$$

$$P^L[\alpha] = \min \left\{ \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{np}[\alpha] \right\} \dots\dots\dots (3)$$

$$P^U[\alpha] = \max \left\{ \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{np}[\alpha] \right\} \dots\dots\dots (4)$$

and fuzzy acceptance probability is as follows:

$$\tilde{p}_a = \left\{ \sum_{d=0}^c \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{\lambda}[\alpha] \right\} = [P^L[\alpha], P^U[\alpha]]$$

$$P^L[\alpha] = \min \left\{ \sum_{d=0}^c \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{\lambda}[\alpha] \right\} \dots\dots\dots (5)$$

$$P^U[\alpha] = \max \left\{ \sum_{d=0}^c \frac{e^{-\lambda} \lambda^d}{d!} \mid \lambda \in \tilde{\lambda}[\alpha] \right\} \dots\dots\dots (6)$$

Acceptance Single Sampling Plan with Fuzzy Parameter using Zero-Inflated Poisson Distribution

When a random sample is drawn from a production process in a continuous stream the observed number of defects in the sample is distributed according to Poisson distribution with parameter $\lambda = np$, which is the average number of defects per unit. Schilling (1982) has pointed out when $n/N \leq 0.10$, n is large, p is small such that $np < 5$, the Poisson distribution is appropriate.

In many practical situations, a Poisson random variable can take the value “0” quite frequently. For instance, when the manufacturing equipment is properly aligned, number of defects may be nearly zero. Under such circumstances, the suitable probability distribution of the number of defects is a zero-inflated Poisson distribution rather than the usual Poisson distribution.

A Zero-Inflated Poisson distribution (ZIP) is defined by the following probability mass function.

$$P(X = x|Z_i, \lambda) = Z_i f(x) + (1 - Z_i) P(X = x|\lambda) \dots\dots\dots (7)$$

where

$$f(x) = \begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{if } x \neq 0 \end{cases} \dots\dots\dots (8)$$

and

$$P(X = x|\lambda) = \frac{e^{-\lambda} \lambda^x}{x!}, \text{ where } x = 0, 1, 2, \dots$$

The above probability mass function can also be expressed as

$$P(X = x|Z_i, \lambda) = \begin{cases} Z_i + (1 - Z_i)e^{-\lambda}, & \text{when } x = 0 \\ (1 - Z_i) \frac{e^{-\lambda} \lambda^x}{x!}, & \text{when } x = 1, 2, \dots, 0 < Z_i < 1, \lambda > 0 \end{cases} \dots\dots\dots (9)$$

In this distribution, Z_i may be termed as the mixing proportion. Z_i and λ are the parameters of the ZIPD. According to McLachlan and Peel (2000), a ZIPD is a special kind of mixture distribution.

The OC function of the ZIP (Z_i, λ) distribution is as follows

$$P_{a(P)} = Z_i + (1 - Z_i)e^{-\lambda} + \sum_{x=1}^{c_0} (1 - Z_i) \frac{e^{-\lambda} \lambda^x}{x!} \dots\dots\dots (10)$$

and fuzzy acceptance probability for P is as follows:

$$P_{ZIP}^L = \min \left\{ Z_i + (1 - Z_i)e^{-\lambda} + (1 - Z_i) \sum_{x=1}^{c_0} \frac{e^{-\lambda} \lambda^x}{x!} \right\} \dots\dots\dots (11)$$

$$P_{ZIP}^U = \max \left\{ Z_i + (1 - Z_i)e^{-\lambda} + (1 - Z_i) \sum_{x=1}^{c_0} \frac{e^{-\lambda} \lambda^x}{x!} \right\} \dots\dots\dots (12)$$

$$\tilde{P}_a = [P_{ZIP}^L(\alpha), P_{ZIP}^U(\alpha)]$$

OC Band with Fuzzy Parameters:

The Operating Characteristic (OC) curve is a key criterion in a sampling plan. It helps determine the probability of accepting or rejecting a lot with a specific number of defective items. The OC curve shows the performance of acceptance sampling plans by plotting the probability of accepting a lot against its quality, which is represented by the proportion of defective items. It aids in selecting plans that effectively reduce risk and demonstrates the discriminating power of the plan.

In a fuzzy environment, the probability of accepting a lot is expressed as a band with upper and lower bounds, based on the fuzzy fraction of defective items. The bandwidth of this band depends on the degree of uncertainty in the proportion parameter. A lower uncertainty results in a narrower bandwidth, and if the proportion parameter has a precise value, the upper and lower bounds converge, resulting in a classic OC curve. By understanding the degree of uncertainty in the proportion parameter and its variation along the horizontal axis, different fuzzy numbers (\tilde{p}) can be obtained, leading to different proportions (p) and the corresponding OC bands.

Example:

The washing machine manufacturing industry is interested in assessing the reliability of its products over time. To achieve this, the company aims to investigate the number of repair incidents per month for a sample of washing machines in use.

In that, if half percent of the products are poorly packaged, major customers inspect 50 items from the available stock before making a purchase decision. If there is a high occurrence of zero defective items, the ZIPD method can be applied. Customers will buy all the products if the sample contains one defective item. If the sample contains more than one defective item, the customers will not make the purchase. Given the proportion of defective products is described linguistically, a fuzzy number $\tilde{p} = (0, 0.005, 0.01)$ is used. Therefore, the probability purchasing would be described in the following:

In the single sampling plan, $n = 50$, $C_0 = 1$, $\tilde{p} = (0, 0.005, 0.01)$ and in the ZIPD, $\tilde{\lambda} = [0, 0.25, 0.5]$, $\tilde{\lambda}[\alpha] = [0.25\alpha, 0.5 - 0.25\alpha]$, then corresponding probability for single sampling plan is obtained to get the probability of acceptance of the plan.

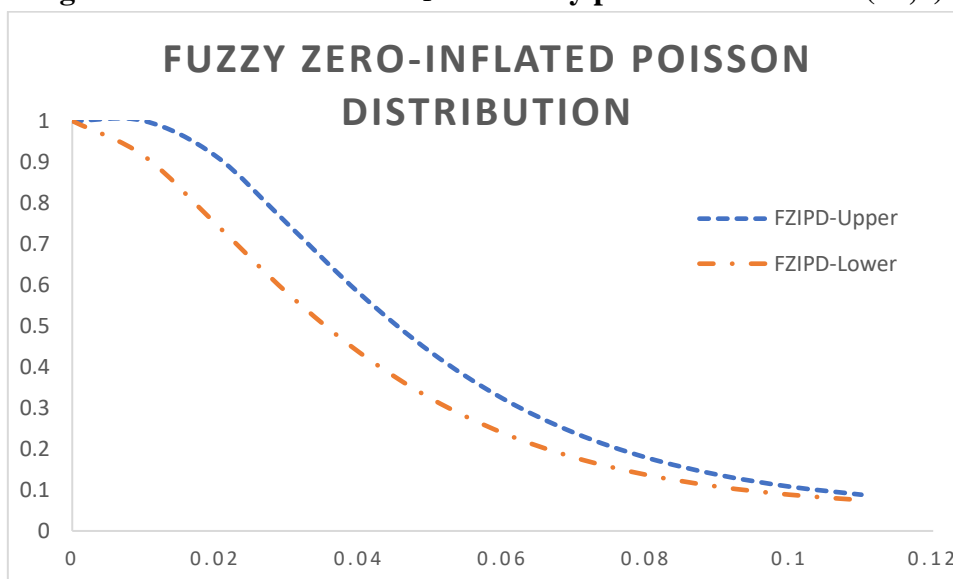
Table 1: Probability of Acceptance for ZIPD_F(n; c₀)

The Table 1 shows the probability of acceptance for zero-inflated Poisson distribution using fuzzy which is calculated for fixed sample size 50 and the acceptance number 1.

Z_i	\tilde{p}	ZIPD _F (50,1)
0.05	[0,0.01]	[1,0.9143]
0.05	[0.01,0.02]	[0.9143,0.7489]
0.05	[0.02,0.03]	[0.7489,0.5799]
0.05	[0.03,0.04]	[0.5799,0.4357]
0.05	[0.04,0.05]	[0.4357,0.3229]
0.05	[0.05,0.06]	[0.3229,0.2391]
0.05	[0.06,0.07]	[0.2391,0.1790]
0.05	[0.07,0.08]	[0.1790,0.1369]
0.05	[0.08,0.09]	[0.1369,0.1080]

0.05	[0.09,0.10]	[0.1080,0.0884]
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Figure 1: OC band for ZIPD_F with fuzzy parameter of ZIPD (50;1)



The above Figure shows the OC band for the ZIPD_F which holds the upper and lower bounds.

Comparison of Sample sizes:

The following table gives the comparison of probability of acceptance of Fuzzy Zero-Inflated Poisson distribution for various \tilde{p} & Z_i with same acceptance number and different sample sizes are as $n=30$ & 50 and $c=2$ and its OC curve is presented in Figure 2.

Table 2: Comparative Probability of Acceptance for Fuzzy ZIPD (30;2) & Fuzzy ZIPD (50;2)

Z_i	\tilde{p}	Pa(P)	
		n=30	n=50
0.05	[0,0.01]	[1,0.9965]	[1,0.9863]
0.05	[0.01,0.02]	[0.9965,0.9780]	[0.9863,0.9237]
0.05	[0.02,0.03]	[0.9780,0.9402]	[0.9237,0.8184]
0.05	[0.03,0.04]	[0.9402,0.8855]	[0.8184,0.6928]
0.05	[0.04,0.05]	[0.8855,0.8184]	[0.6928,0.5666]
0.05	[0.05,0.06]	[0.8184,0.7440]	[0.5666,0.4520]
0.05	[0.06,0.07]	[0.7440,0.6671]	[0.4520,0.3548]
0.05	[0.07,0.08]	[0.6671,0.5912]	[0.3548,0.2762]
0.05	[0.08,0.09]	[0.5912,0.5189]	[0.2762,0.2149]
0.05	[0.09,0.10]	[0.5189,0.4520]	[0.2149,0.1684]
0.05	[0.10,0.11]	[0.4520,0.3914]	[0.1684,0.1339]

0.05	[0.11,0.12]	[0.3914,0.3376]	[0.1339,0.1088]
0.05	[0.12,0.13]	[0.3376,0.2904]	[0.1088,0.0908]
0.05	[0.13,0.14]	[0.2904,0.2497]	[0.0908,0.0781]
0.05	[0.14,0.15]	[0.2497,0.2148]	[0.0781,0.0692]
0.05	[0.15,0.16]	[0.2148,0.1854]	[0.0692,0.0630]

Figure: 2

This Figure shows the OC bands for sample sizes $n=30$ and $n=50$ with acceptance number $c=2$. Indicating that OC bands are convex with one acceptance number and this leads to a quick reduction of fuzzy probability of acceptance for proportion of defective items with small fuzzy numbers, and it will be more the increase of n .

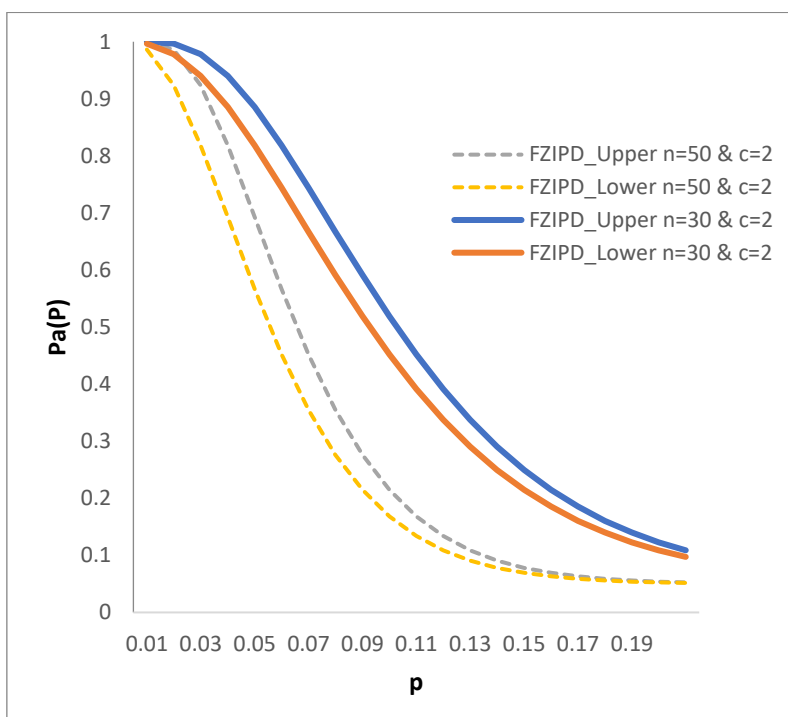


Table 3: Comparative Probability of Acceptance of ZIPD with Poisson distribution using Fuzzy

The following Table shows the Probability of acceptance for various \tilde{p} & Z_i for fuzzy Poisson distribution and fuzzy ZIPD with parameters $n=50, c=2$.

Z_i	\tilde{p}	Fuzzy Poisson (50,2)	Fuzzy ZIPD (50,2)
0.05	[0,0.01]	[1,0.9856]	[1,0.9863]
0.05	[0.01,0.02]	[0.9856,0.9196]	[0.9863,0.9237]
0.05	[0.02,0.03]	0.9196,0.8088]	[0.9237,0.8184]
0.05	[0.03,0.04]	[0.8088,0.6766]	[0.8184,0.6928]
0.05	[0.04,0.05]	[0.6766,0.5438]	[0.6928,0.5666]
0.05	[0.05,0.06]	[0.5438,0.4231]	[0.5666,0.4520]
0.05	[0.06,0.07]	[0.4231,0.3208]	[0.4520,0.3548]
0.05	[0.07,0.08]	[0.3208,0.2381]	[0.3548,0.2762]

0.05	[0.08,0.09]	[0.2381,0.1735]	[0.2762,0.2149]
0.05	[0.09,0.10]	[0.1735,0.1246]	[0.2149,0.1684]
0.05	[0.10,0.11]	[0.1246,0.0883]	[0.1684,0.1339]
0.05	[0.11,0.12]	[0.0883,0.0619]	[0.1339,0.1088]
0.05	[0.12,0.13]	[0.0619,0.0430]	[0.1088,0.0908]
0.05	[0.13,0.14]	[0.0430,0.0296]	[0.0908,0.0781]
0.05	[0.14,0.15]	[0.0296,0.0202]	[0.0781,0.0692]
0.05	[0.15,0.16]	[0.0202,0.0137]	[0.0692,0.0630]

Figure: 3

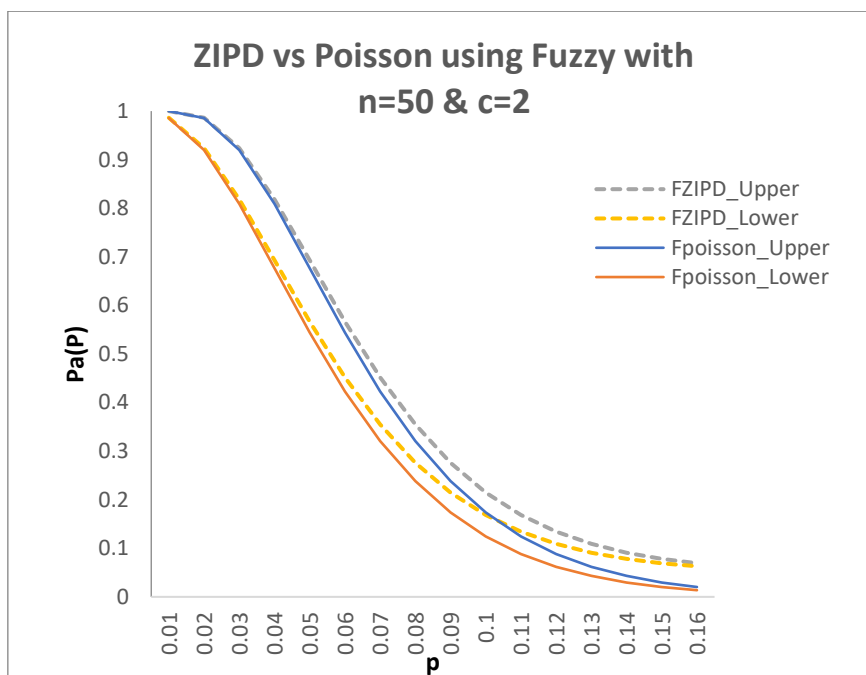


Figure 3 illustrates the probability of acceptance for the fuzzy zero-inflated Poisson distribution and the fuzzy Poisson distribution, using the same sample size and acceptance number. The figure demonstrates that the two bands closely approximate each other. Ultimately, this suggests that the OC band using the fuzzy ZIPD provides an optimal approximation for the OC band using the Fuzzy Poisson distribution. Therefore, compared to the Fuzzy Poisson distribution, the Fuzzy ZIPD generally yields a higher probability of acceptance.

Conclusion:

In a well-equipped production process, the majority of products meet specified quality standards, often resulting in a higher frequency of zero non-conformities during sampling inspection. The zero-inflated model is the appropriate probability distribution for the number of non-conformities per product manufactured under such conditions. This paper figures out the Probability of acceptance for Single Sampling Plan using Fuzzy ZIPD for the desired parameters and also assessed for different sample sizes with fixed acceptance number. Under Fuzziness environment the Zero Inflated Poisson Distribution is compared with Poisson distribution with single sampling plan as the base line distribution for the specified parameters to highlight its advantages and significances. Accordingly, the Probability of acceptance for Fuzzy ZIPD is found to be efficient than Fuzzy Poisson Distribution. In the shop floor situations when there is more uncertainty the fuzzy ZIPD is more beneficial for both consumer and producer.

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