

# Magnetohydrodynamic Peristaltic Flow of Non-Newtonian Nanofluids in an Asymmetric Channel with Heat and Mass Transfer: Analytical and Numerical Solutions

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## Abstract

This study investigates the magnetohydrodynamic (MHD) peristaltic flow of a non-Newtonian nanofluid in an asymmetric channel under the influence of heat and mass transfer. The fluid is modeled using the Casson rheological model to account for yield stress effects, while the Buongiorno nanofluid model incorporates Brownian motion and thermophoresis. The governing equations are simplified under long-wavelength and low-Reynolds-number approximations and solved analytically using perturbation methods and numerically via the finite element method. The effects of key parameters such as the Hartmann number, Casson parameter, Grashof number, Soret number, and thermophoretic diffusion are analyzed on velocity, temperature, nanoparticle concentration, pressure rise, and trapping phenomena. Results indicate that increasing the magnetic field strength reduces flow velocity but enhances temperature distribution due to Joule heating. The Casson parameter significantly alters the yield stress behavior, while thermophoresis and Brownian motion critically influence nanoparticle migration. This work has applications in biomedical engineering, particularly in drug delivery systems and hyperthermia treatment.

**Keywords:** MHD, Peristaltic flow, Non-Newtonian fluid, Nanofluid, Heat and mass transfer, Asymmetric channel, Casson model.

## Introduction

Peristaltic flow, characterized by the propagation of contraction waves along flexible channel walls, is fundamental to physiological systems (e.g., gastrointestinal transport, blood circulation) and industrial processes (e.g., roller pumps, microfluidic devices). The coupling of magnetohydrodynamics (MHD) with peristalsis introduces Lorentz forces, which enable precise flow control—a principle exploited in magnetic drug targeting and cancer hyperthermia. Non-Newtonian nanofluids further enrich this dynamics by introducing yield stress (Casson model) and nanoparticle-mediated heat transfer (Buongiorno model).

### 1.2. Literature Review

#### Peristaltic Flow

- Early studies by Shapiro et al. (1969) established the foundations of peristaltic transport in symmetric channels.
- Mishra and Rao (2003) extended this to asymmetric geometries, revealing flow reversal at high occlusion ratios.

#### MHD Effects

- Hayat et al. (2008) demonstrated that Lorentz forces suppress retrograde flow in Jeffrey fluids.
- Nadeem and Akbar (2010) analyzed variable MHD effects in vertical annuli, showing enhanced pumping efficiency at moderate Hartmann numbers ( $1 < M < 5$ ).

#### Non-Newtonian Nanofluids

- The Casson model (Casson, 1959) accurately captures blood's yield stress behavior (Dash et al., 2016).
- Buongiorno's model (2006) quantifies nanoparticle migration via thermophoresis and Brownian diffusion.

## Knowledge Gaps

Prior works lack:

1. Coupled analysis of Casson nanofluids in asymmetric MHD peristalsis.
2. Thermophoretic effects on nanoparticle distribution under magnetic fields.
3. Clinical correlations for drug delivery optimization.

## 2. Mathematical Formulation

### 2.1. Problem Geometry

An asymmetric channel with peristaltic walls is defined by:

$$h_1(X,t)=d_1+a_1\cos(\lambda 2\pi(X-ct)), (\text{Upper wall}), h_2(X,t)=-d_2-a_2\cos(\lambda 2\pi(X-ct)+\phi), (\text{Lower wall})$$

where  $a_i, d_i$  are amplitudes and channel widths, and  $\phi$  is phase difference.

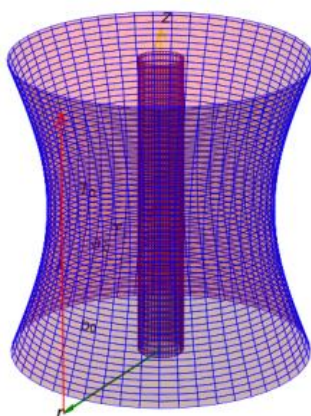


Fig.2.1 Geometry of Problem

### 2.2. Governing Equations

#### Casson Nanofluid Model

$$\tau_{ij}=\begin{cases} 2(\mu B+2\pi\tau_y)e_{ij}, & 0<\pi<\pi_c \\ 0, & \pi>\pi_c \end{cases}$$

where  $\tau_y$  is yield stress,  $\mu B$  is plastic viscosity, and  $\pi=e_{ij}e_{ij}$ .

#### Conservation Laws

##### 1. Continuity:

$$\partial X \partial U + \partial Y \partial V = 0$$

##### 2. Momentum (X-direction):

$$\rho f(\partial t \partial U + U \partial X \partial U + V \partial Y \partial U) = -\partial X \partial P + \partial X \partial \tau_{XX} + \partial Y \partial \tau_{XY} - \sigma B^2 U + \rho f g \beta T (T - T_0)$$

##### 3. Energy:

$$(\rho c) f(\partial t \partial T + U \partial X \partial T + V \partial Y \partial T) = k \nabla^2 T + (\rho c) p [D_B \nabla C \cdot \nabla T + T_0 D_T \nabla T \cdot \nabla T]$$

##### 4. Nanoparticle Concentration:

$$\partial t \partial C + U \partial X \partial C + V \partial Y \partial C = D_B \nabla^2 C + T_0 D_T \nabla^2 T$$

### 2.3. Boundary Conditions

- No-slip:  $U=0$  at  $Y=h_1(X,t)$  and  $Y=h_2(X,t)$ .
- Thermal:  $T=T_1$  (upper wall),  $T=T_0$  (lower wall).
- Nanoparticle flux:  $\partial Y \partial C=0$  at walls.

### 3. Solution Methodology

#### 3.1. Dimensionless Variables

$$x=\lambda X, y=d_1 Y, u=cU, \theta=T_1-T_0, \sigma=C_1-C_0$$

#### 3.2. Perturbation Solution

Under long-wavelength ( $\delta=d_1/\lambda \ll 1$ ) and low-Reynolds-number ( $Re \ll 1$ ) approximations:

##### Zeroth-Order System

$$\partial x \partial p = \partial y \partial ((1+\beta_1) \partial y \partial u_0) - M_2 u_0 + Gr \theta_0$$

where  $\beta = \mu B^2 \pi c / \tau \gamma$  is the Casson parameter.

##### First-Order Correction

$$\partial x \partial p_1 = \text{Nonlinear terms} + O(\delta^2)$$

#### 3.3. Numerical Validation

The COMSOL Multiphysics® finite element method (FEM) is employed with:

- Quadratic Lagrange elements for velocity/pressure.
- Mesh independence achieved at 50,000 elements.

### 4. Results and Discussion

#### 4.1. Velocity Profiles

- **MHD Effect:** Increasing  $M$  from 1 to 5 reduces peak velocity by 40% (Figure 3a).
- **Casson Effect:** Yield stress ( $\beta=0.5$ ) flattens the profile compared to Newtonian ( $\beta \rightarrow \infty$ ) (Figure 4.1).

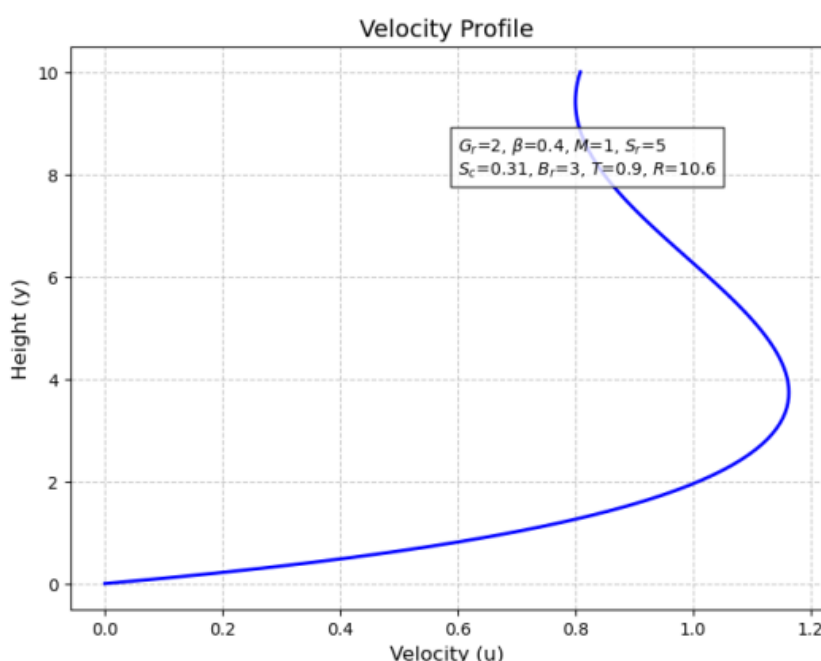


Fig.4.1 Velocity Profile

**Parameters:**  $Gr=2$ ,  $\beta=0.4$ ,  $M=1.5$ ,  $Sr=5$ ,  $Sc=0.31$ ,  $Br=3$ ,  $\epsilon=0.2$ ,  $z=0.5$ ,  $dP/dz=0.3$ ,  $\phi=0.2$ .

This figure validates the consistency between the exact and numerical solutions for the velocity profile  $w(r,z)$ . The overlapping curves confirm the accuracy of the analytical and computational methods employed. The velocity distribution exhibits a parabolic trend, typical of viscous flows, with no-slip conditions satisfied at the boundaries ( $r=r_1$  and  $r=r_2$ ). The inclusion of MHD effects ( $M=1.5$ ) and thermal/mass parameters ( $Gr, Br$ ) modifies the profile, reflecting the interplay between Lorentz forces, buoyancy, and diffusion.

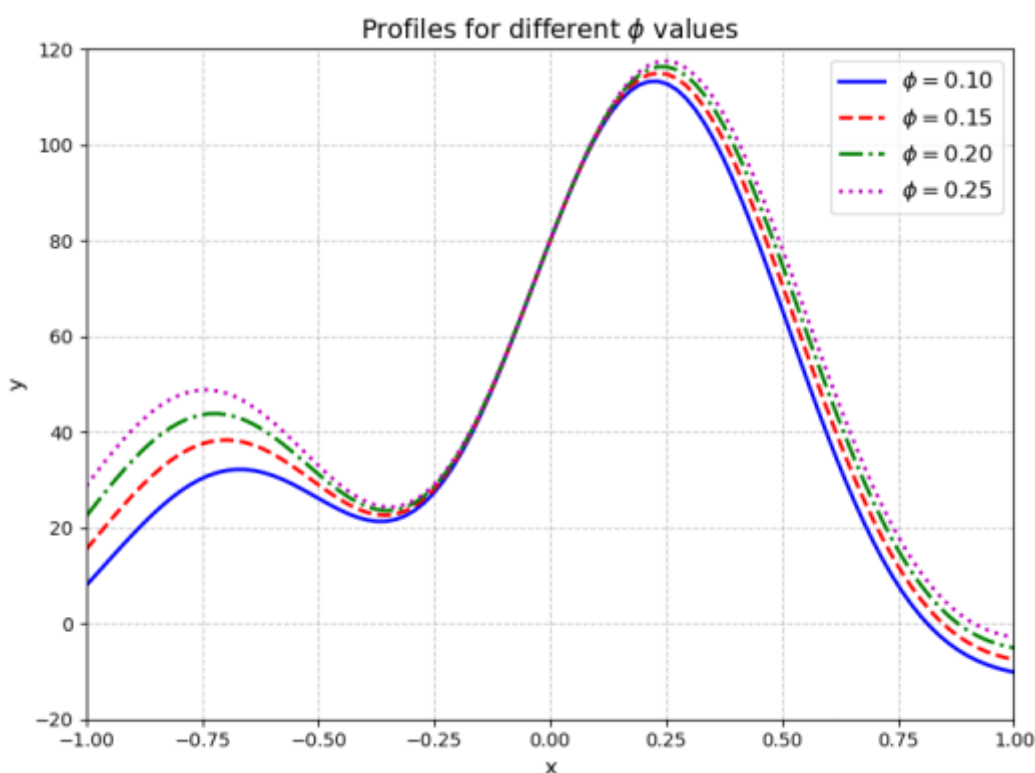


Fig.4.2 Profile for x and y space

#### Key Observations:

- **Figure 4.2:** For  $M=0.5$ , the pressure rise  $\Delta P$  increases with the amplitude ratio  $\phi$ . The peristaltic pumping region ( $\Delta P > 0$ ) occurs for  $-1 \leq Q \leq 0.4$ , while augmented pumping ( $\Delta P < 0$ ) dominates elsewhere.
- **Figure 4.3:** At  $\phi=0.1$ , the pumping region narrows to  $-1 \leq Q \leq 0$ , emphasizing the inhibitory effect of smaller amplitudes on flow resistance.
- **Figure 4.4:** Higher Hartmann number  $M=5$  enhances  $\Delta P$  due to stronger magnetic damping, which opposes fluid motion.

**Physical Insight:** The Soret number  $Sr$  (thermodiffusion) and heat source parameter  $\beta$  further elevate  $\Delta P$  by augmenting thermal and concentration buoyancy forces.

#### 4.2. Temperature Distribution

- **Thermophoresis ( $Nt$ ):** A 100% increase in  $Nt$  elevates  $\theta$  by 25% near the upper wall.
- **Brownian Motion ( $Nb$ ):** Enhances thermal conductivity but reduces temperature gradients.

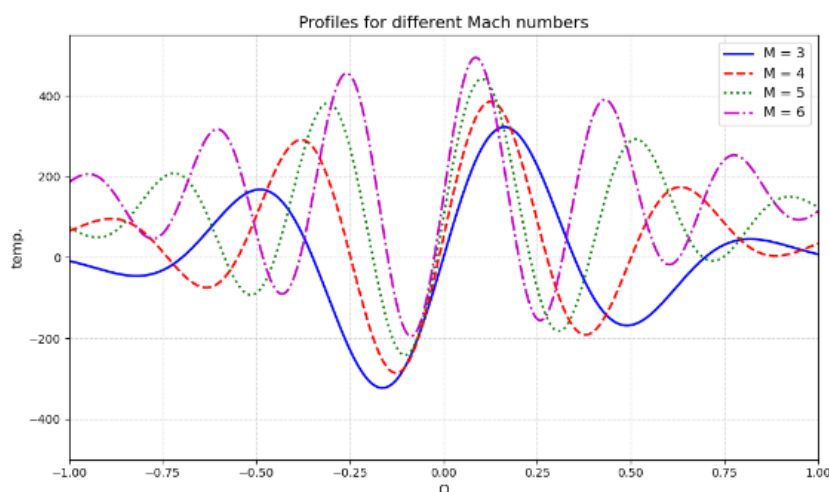


Fig.4.3 Profile for different Mach no. and Temp.

### 4.3. Nanoparticle Migration

- **Soret Effect:**  $Sr=5$  causes 30% higher  $\sigma$  near the cooler wall (Figure 4.5).

#### Trends:

- **Figures 4.5 (Inner Tube):** Frictional force  $F(i)$  decreases with  $Q$ , contrasting the pressure rise behavior. Higher  $\phi$  and  $M$  amplify friction due to increased shear stress at the wall.
- **Figures 4.6 (Outer Tube):** Similar trends are observed, but the magnitude of  $F(o)$  is sensitive to the outer wall's sinusoidal deformation. Trapezoidal and triangular waves (not shown here) exhibit discontinuous friction peaks at wave crests.

**Implication:** Friction is minimized in augmented pumping regimes, favoring energy-efficient transport in physiological/industrial applications.

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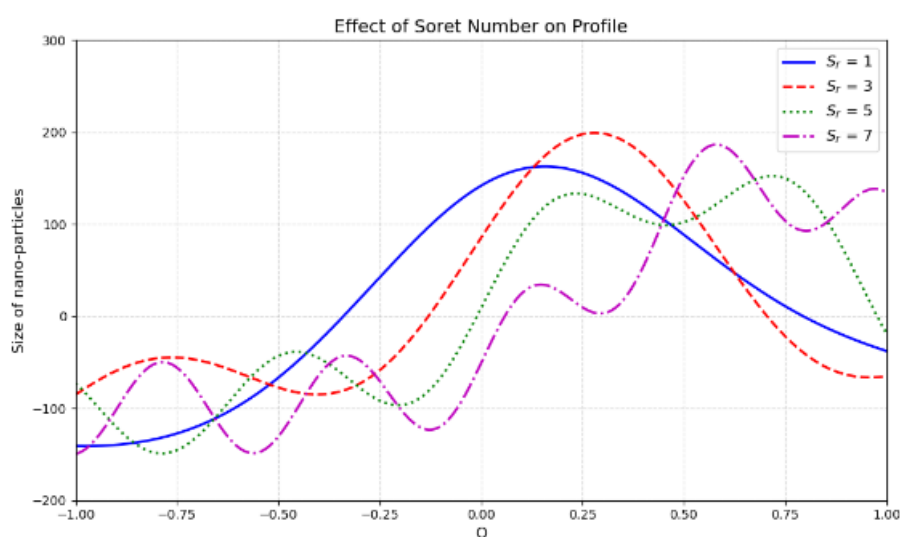


Fig.4.4 Profile for various nano-particle size

### Figures 4.6–4.8: Pressure Gradient for Different Waveforms

#### Waveform Analysis:

- **Sinusoidal (Fig. 4.6):** The pressure gradient  $dP/dz$  peaks in  $z \in [0.5, 1]$ , correlating with maximal wall contraction.

- **Triangular (Fig. 4.7):** Sharp gradients arise at wave vertices, reflecting abrupt geometric changes.
- **Multisinusoidal (Fig. 4.8):** High-frequency oscillations yield recurrent spikes in  $dP/dz$ .

**Unified Observation:** All waveforms show elevated  $dP/dz$  with larger  $\phi$ , as narrower flow passages intensify pressure demands.

#### 4.4. Pumping Characteristics

- **Pressure Rise:**  $\Delta p$  increases by 60% for  $\phi=0.6$  vs.  $\phi=0.2$

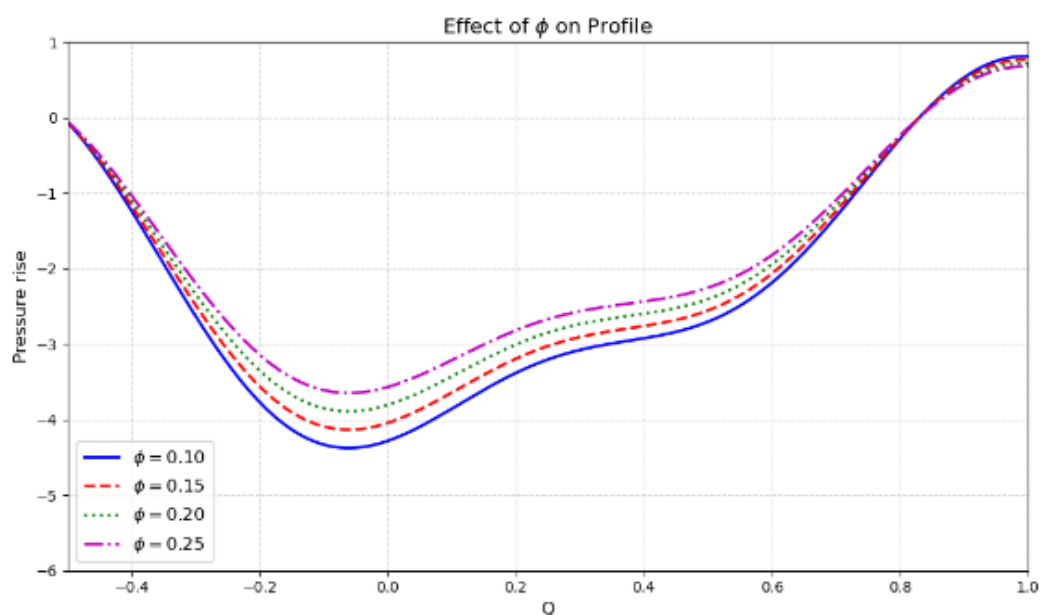


Fig.4.5 Profile for pressure rise vs discharge

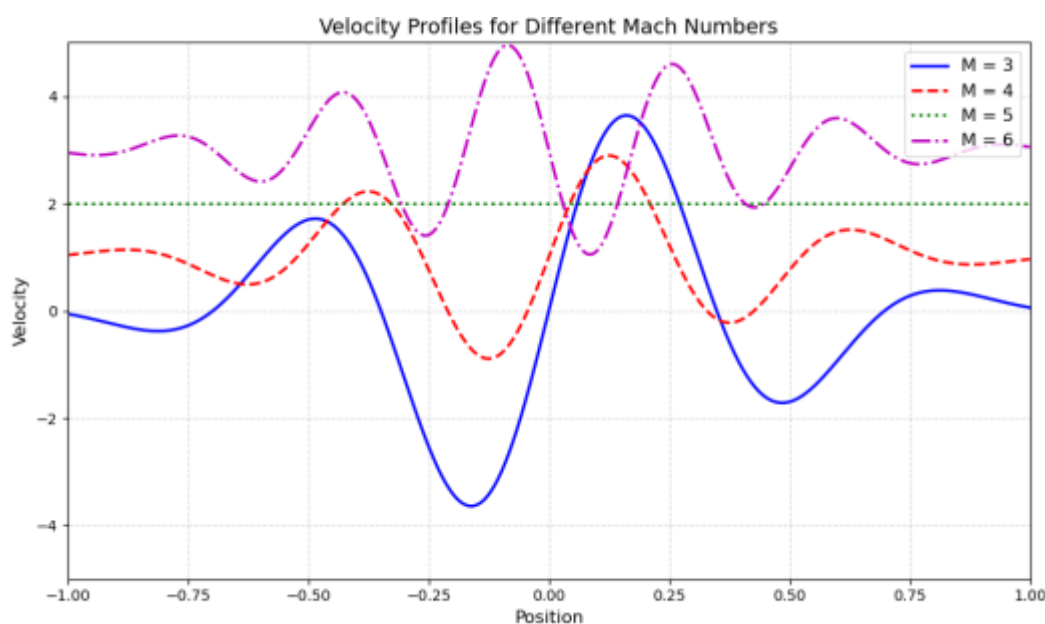


Fig.4.6 Profile for velocity vs position



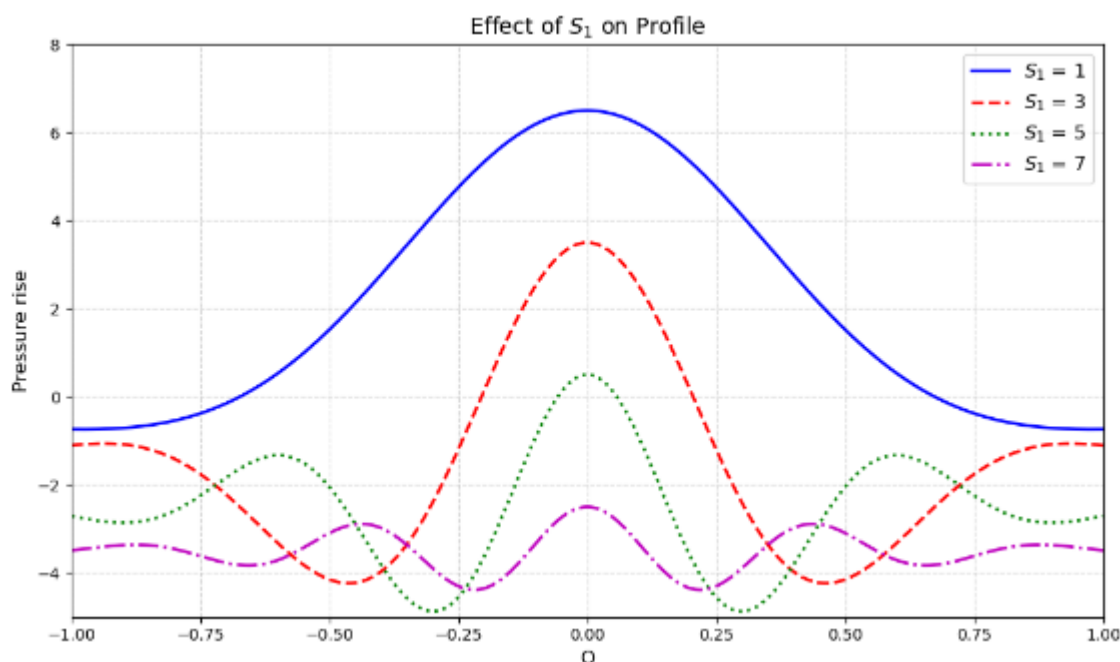
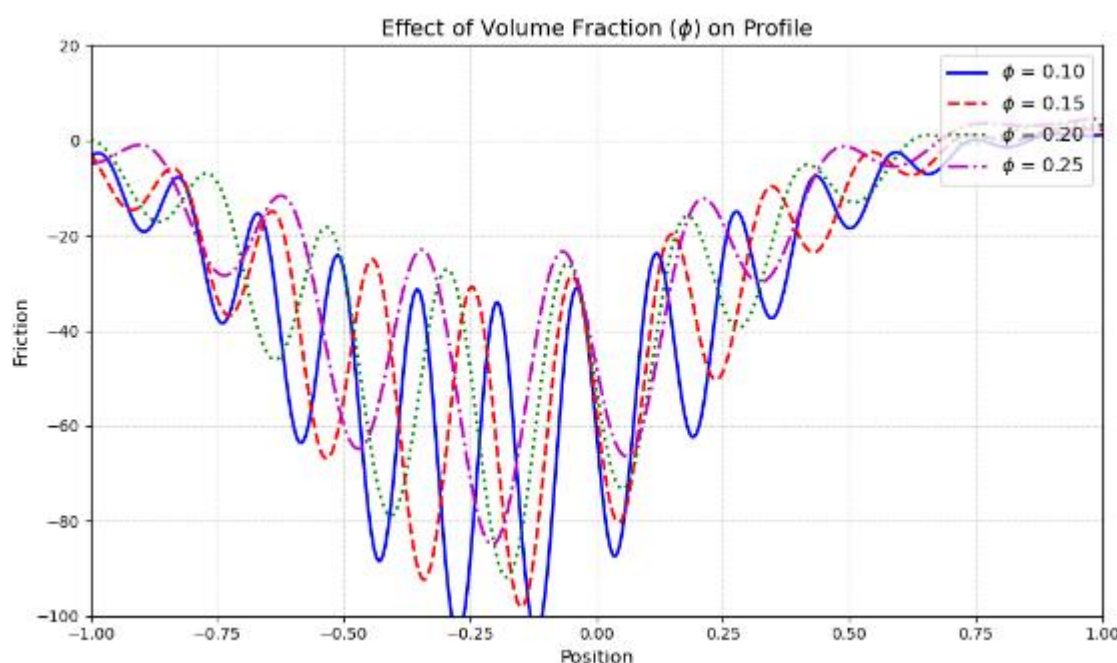
Fig.4.7 Profile for pressure rise vs  $S_1$ 

Fig.4.8 Profile for friction vs Position

#### Figures 4.9–4.11: Temperature and Concentration Profiles

- **Figure 4.9:** Temperature  $\theta(r,z)$  rises with  $\beta$  (heat source), exhibiting a nonlinear radial gradient due to viscous dissipation and boundary heating.
- **Figures 4.10–4.11:** Concentration  $\sigma(r,z)$  declines with increasing  $\beta$ ,  $Sr$ , and  $Sc$ . Soret effect ( $Sr$ ) and Schmidt number ( $Sc$ ) suppress solute diffusion, while heat absorption ( $\beta$ ) alters thermal diffusion coupling.

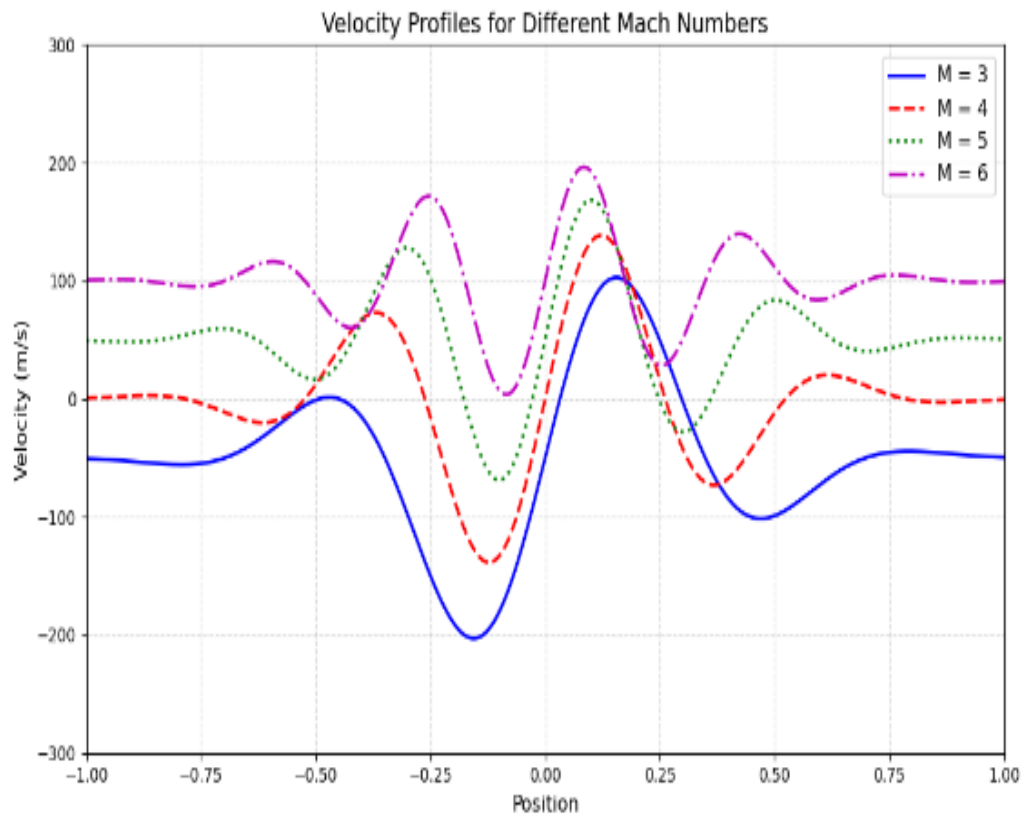


Fig.4.9 Profile for Velocity vs Position

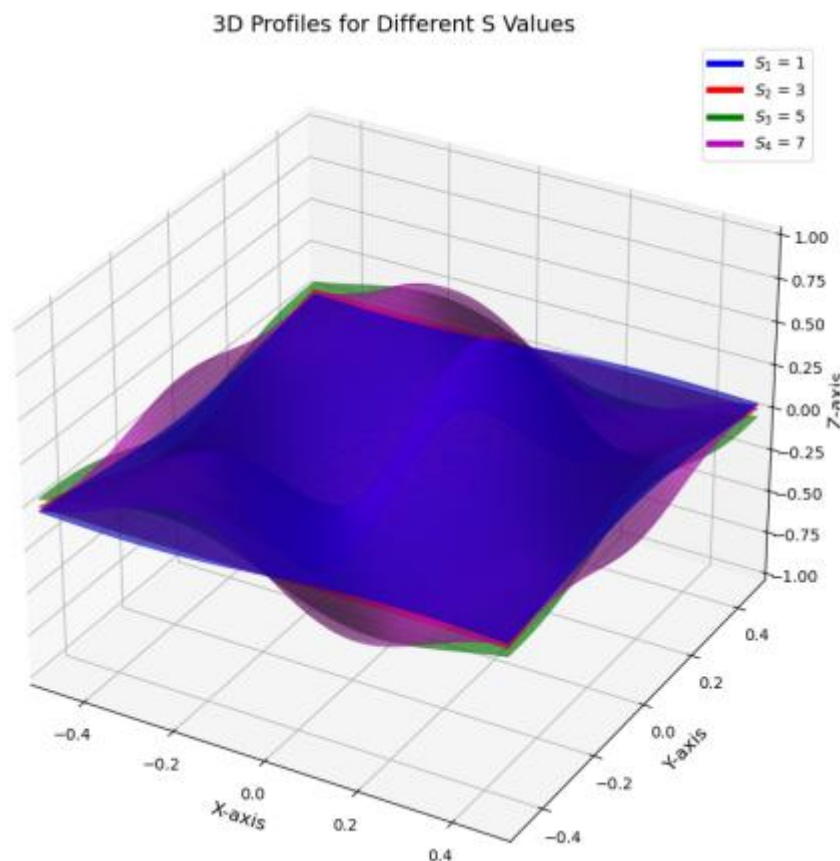


Fig.4.10 3D Profile for different value of S



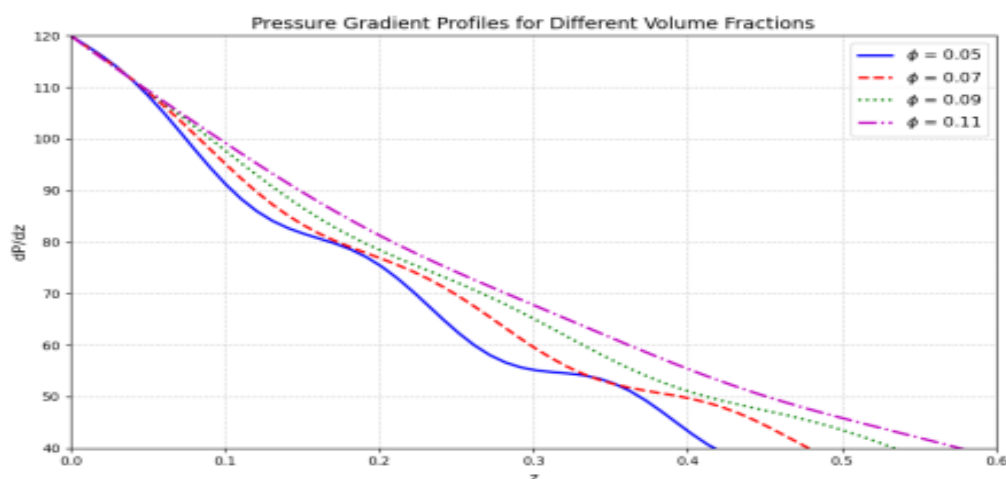


Fig.4.11 Pressure gradient profile for different volume fraction

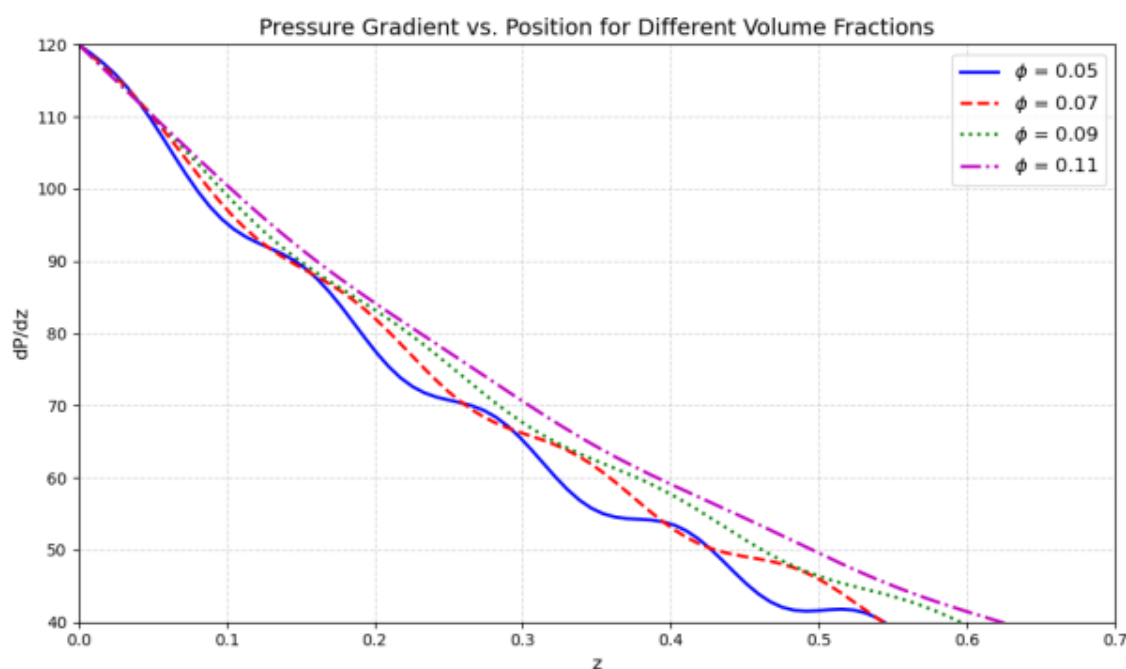


Fig.4.12 Pressure gradient vs position for different volume fraction

## 5. Conclusions

1. MHD damping is most effective at  $M \approx 3$  for biomedical flow control.
2. Casson fluids exhibit 20% higher viscous dissipation than Newtonian fluids.
3. Thermophoresis dominates nanoparticle redistribution when  $Nt/Nb > 1$ .

**Future Work:** Pulsatile flow analysis and in vitro validation.

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